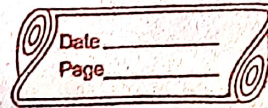


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## Vector Calculus

Theorem →

Prove that

$$\text{div}(\vec{u} + \vec{v}) = \text{div} \vec{u} + \text{div} \vec{v}$$

$$\text{or } \nabla \cdot (\vec{u} + \vec{v}) = \nabla \cdot \vec{u} + \nabla \cdot \vec{v}$$

Proof

$$\text{LHS} = \text{Div}(\vec{u} + \vec{v}) = \nabla \cdot (\vec{u} + \vec{v})$$

$$= \left( \vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z} \right) \cdot (\vec{u} + \vec{v})$$

$$= \vec{i} \cdot \frac{\partial}{\partial x} (\vec{u} + \vec{v}) + \vec{j} \cdot \frac{\partial}{\partial y} (\vec{u} + \vec{v})$$

$$+ \vec{k} \cdot \frac{\partial}{\partial z} (\vec{u} + \vec{v})$$

$$= \vec{i} \cdot \left[ \frac{\partial \vec{u}}{\partial x} + \frac{\partial \vec{v}}{\partial x} \right] + \vec{j} \cdot \left[ \frac{\partial \vec{u}}{\partial y} + \frac{\partial \vec{v}}{\partial y} \right]$$

$$+ \vec{k} \cdot \left[ \frac{\partial \vec{u}}{\partial z} + \frac{\partial \vec{v}}{\partial z} \right]$$

$$= \left( \vec{i} \cdot \frac{\partial \vec{u}}{\partial x} + \vec{j} \cdot \frac{\partial \vec{u}}{\partial y} + \vec{k} \cdot \frac{\partial \vec{u}}{\partial z} \right)$$

$$+ \left( \vec{i} \cdot \frac{\partial \vec{v}}{\partial x} + \vec{j} \cdot \frac{\partial \vec{v}}{\partial y} + \vec{k} \cdot \frac{\partial \vec{v}}{\partial z} \right)$$

$$= \left( \vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z} \right) \cdot \vec{u} + \left( \vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z} \right) \cdot \vec{v}$$

$$= \nabla \cdot \vec{u} + \nabla \cdot \vec{v}$$

$$= \text{Div} \vec{u} + \text{Div} \vec{v} = \text{RHS proved}$$

Theorem

Prove that  $\text{curl}(\vec{u} + \vec{v}) = \text{curl} \vec{u} + \text{curl} \vec{v}$

or  $\nabla \times (\vec{u} + \vec{v}) = \nabla \times \vec{u} + \nabla \times \vec{v}$

Proof

LHS =  $\text{curl}(\vec{u} + \vec{v}) = \nabla \times (\vec{u} + \vec{v})$

$= (\vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z}) \times (\vec{u} + \vec{v})$

$= \vec{i} \times \frac{\partial}{\partial x} (\vec{u} + \vec{v}) + \vec{j} \times \frac{\partial}{\partial y} (\vec{u} + \vec{v}) + \vec{k} \times \frac{\partial}{\partial z} (\vec{u} + \vec{v})$

$= \vec{i} \times \left[ \frac{\partial \vec{u}}{\partial x} + \frac{\partial \vec{v}}{\partial x} \right] + \vec{j} \times \left( \frac{\partial \vec{u}}{\partial y} + \frac{\partial \vec{v}}{\partial y} \right)$

$+ \vec{k} \times \left( \frac{\partial \vec{u}}{\partial z} + \frac{\partial \vec{v}}{\partial z} \right)$

$= \left( \vec{i} \times \frac{\partial \vec{u}}{\partial x} + \vec{i} \times \frac{\partial \vec{v}}{\partial x} \right) + \left( \vec{j} \times \frac{\partial \vec{u}}{\partial y} + \vec{j} \times \frac{\partial \vec{v}}{\partial y} \right)$

$+ \left( \vec{k} \times \frac{\partial \vec{u}}{\partial z} + \vec{k} \times \frac{\partial \vec{v}}{\partial z} \right)$

$= \left( \vec{i} \times \frac{\partial \vec{u}}{\partial x} + \vec{j} \times \frac{\partial \vec{u}}{\partial y} + \vec{k} \times \frac{\partial \vec{u}}{\partial z} \right)$

$+ \left( \vec{i} \times \frac{\partial \vec{v}}{\partial x} + \vec{j} \times \frac{\partial \vec{v}}{\partial y} + \vec{k} \times \frac{\partial \vec{v}}{\partial z} \right)$

$= \left( \vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z} \right) \times \vec{u}$

$+ \left( \vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z} \right) \times \vec{v}$

$= \nabla \times \vec{u} + \nabla \times \vec{v}$

$= \text{curl} \vec{u} + \text{curl} \vec{v} = \text{RHS}$

Proved